Introduction

The *College and Career Readiness Standards for Mathematics* consist of three interconnected parts: a Standard for Mathematical Practice, ten Standards for Mathematical Content, and a set of Example Tasks.

The Standard for Mathematical Practice has six Core Practices that describe the way proficient students approach mathematics. Proficient students attend to precision, construct viable arguments, make sense of complex problems and persevere in solving them, look for hidden structure, note regularity in repeated reasoning, and use technology intelligently. This approach to mathematics is an essential part of being ready for college and career.

The Standards for Mathematical Content form the backbone of this document. Each of these ten standards consists of Core Concepts, Core Skills, and a description of the student's Coherent Understanding. Students who encounter the subject with a focus on coherence will be better able to learn more mathematics at a deeper level and be better able to access and apply the mathematics they know. The ten Standards for Mathematical Content pull together topics previously studied and look ahead toward topics in further coursework and training programs.

The Standards for Mathematical Content are designed to draw greater attention to powerful organizing principles in mathematics, such as functional relationships or the laws of arithmetic. They also allow important distinctions to be made more clearly, such as that between Expressions and Equations. And they surface the deep connections that often underlie mathematical coherence, such as the blending of algebra with geometry represented by Coordinates. These ten are not categories or buckets of topics to cover; they are standards. They describe the coherence students need and deserve as they go forward to their mathematical futures.

The third component of the *College and Career Readiness Standards for Mathematics* is a Web-based collection of Example Tasks that exemplifies the variety of performances required. High standards demand that students *use* their knowledge, skills and good practices to solve problems from a variety of contexts, both within mathematics and from the world outside. Example Tasks exemplify the range and variety of use that is expected. Teachers and designers of curriculum and assessment will find in the collection of examples a guide to what these standards mean. Over time, the collection of tasks will grow.

Together, these three components establish an evidence-based standard for college and career readiness. The *College and Career Readiness Standards for Mathematics* have been created with attention to the expectations of the highest achieving countries. They have focus and depth, emphasizing the understanding of and connections among topics that are most important for success regardless of a student's pathway after reaching these standards.

A primary goal of developing these standards is to enable students to achieve *mathematical proficiency* (see sidebar). Students are expected to understand the knowledge described in the Core Concepts and in the Coherent Understandings at a depth that enables them to reason with that knowledge—to analyze, interpret and evaluate mathematical problems, make deductions, and justify results. The Core Skills are meant to be used strategically and adaptively to solve problems. Students' knowledge and skills come to life and take their value when melded with the ways they approach mathematics—as described by the Core Practices.

The specific verbs used to describe concepts and skills in these standards are not meant to limit or indicate levels of any taxonomy. Although using verbs to indicate levels of depth has been a common practice in this country's standards writing, high performing nations do not use verbs in this way. They describe depth and practices first in separate sections of their syllabi. We have adopted the high performing countries' practice of focusing on a clear statement of what mathematics should be learned when writing standards for knowledge and skills.

Instruction, curriculum and assessment designed to achieve these standards should range over all strands of proficiency in *Adding It Up*, all depths of knowledge in Norman L. Webb's Depth of Knowledge taxonomy, all levels of Bloom's Taxonomy, and all levels of cognitive demand. In the Core Skills and Core Practices we have sometimes used terms like "explore" to indicate a lighter treatment with a goal of awareness and experience rather than proficiency. We have used Example Tasks to show the depth of knowledge and deployment of skills expected.

From *Adding it up: Helping children learn mathematics* (National Research Council, 2001, p. 116):

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully. Mathematical proficiency, as we see it, has five components, or strands:

conceptual understanding—comprehension of mathematical concepts, operations, and relations

procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

strategic competence—ability to formulate, represent, and solve mathematical problems

adaptive reasoning—capacity for logical thought, reflection, explanation, and justification

productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These standards are measurable; that is, they are observable and verifiable through the broad spectrum of student performances that may be assessed during classroom observation, school-based examinations and large-scale testing. The *College and Career Readiness Standards for Mathematics* can guide the development of assessment frameworks that distribute the assessment responsibilities across multiple levels of the educational system: state, district, school and teacher.

Students reaching these levels will be prepared for non-remedial college mathematics courses and will be prepared for training programs for career-level jobs; however, the *College and Career Readiness Standards for Mathematics* should not be construed as grade twelve exit standards. Students interested in STEM fields, and those who wish to go beyond for other reasons, will need to reach these standards before their senior year in order to have time to include additional mathematics. A number of pathways for advanced learning are possible and may be integrated throughout the high school experience and beyond.

The Common Core State Standards Initiative

The *College and Career Readiness Standards for Mathematics* will anchor the next phase of the Common Core State Standards Initiative: development of K–12 Mathematics Standards. Those K–12 Standards are in turn expected to guide the development of a next generation of assessments, developed collaboratively by multiple states. The K–12 Mathematics Standards will serve as a guide and tool for aligning instruction, curriculum, assessment, teacher supports, and systems of accountability. To ensure alignment, the Standard for Mathematical Practice, the Standards for Mathematical Content, and the Example Tasks should all be taken into account.

Overview of the Mathematical Practice Standard

Attend to precision.
Construct viable arguments.
Make sense of complex problems and persevere in solving them.
Look for structure.
Look for and express regularity in repeated reasoning.
Make strategic decisions about the use of technological tools.

Overview of the Mathematical Content Standards

Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations on numbers and extend to operations in algebra.

Quantity. A quantity is an attribute of an object or phenomenon that can be specified using a number and a unit, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

Expressions. Expressions use numbers, variables and operations to describe computations. The rules of arithmetic, the use of parentheses and the conventions about order of operations assure that the computation has a well-determined value.

Equations. An equation is a statement that two expressions are equal. Solutions to an equation are the values of the variables in it that make it true.

Functions. Functions model situations where one quantity determines another. Because nature and society are full of dependencies, functions are important tools in the construction of mathematical models.

Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

Shape. From only a few axioms, the deductive method of Euclid generates a rich body of theorems about geometric objects, their attributes and relationships.

Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Probability. Probability assesses the likelihood of an event in a situation that involves randomness. It quantifies the degree of certainty that an event will happen as a number from 0 through 1.

Statistics. Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability in the data.

How Evidence Informed Decisions in Drafting the Standards

The Common Core State Standards Initiative builds on a generation of standards efforts led by states and national organizations. On behalf of the states, we have taken a step toward the next generation of standards that are aligned to college- and career-ready expectations and are internationally benchmarked. These standards are grounded in evidence from many sources that shows that the next generation of standards in mathematics must be focused on deeper, more thorough understanding of more fundamental mathematical ideas and higher mastery of these fewer, more useful skills.

The evidence that supports this new direction comes from a variety of sources. International comparisons show that high performing countries focus on fewer topics and that the U.S. curriculum is "a mile wide and an inch deep." Surveys of college faculty show the need to shift away from high school courses that merely survey advanced topics, toward courses that concentrate on developing an understanding and mastery of ideas and skills that are at the core of advanced mathematics. Reviews of data on student performance show the large majority of U.S. students are not mastering the mile wide list of topics that teachers cover.

The evidence tells us that in high performing countries like Singapore, the gap between what is taught and what is learned is relatively smaller than in Malaysia or the U.S. states. Malaysia's standards are higher than Singapore's, but their performance is much lower. One could interpret the narrower gap in Singapore as evidence that they actually use their standards to manage instruction; that is, Singapore's standards were set within the reach of hard work for their system and their population. Singapore's Ministry of Education flags its webpage with the motto, "Teach Less, Learn More." We accepted the challenge of writing standards that could work that way for U.S. teachers and students: By providing focus and coherence, we could enable more learning to take place at all levels.

However, a set of standards cannot be simplistically "derived" from any body of evidence. It is more accurate to say that we used evidence to inform our decisions. A few examples will illustrate how this was done.

For example, systems of linear equations are covered by all states, yet students perform surprisingly poorly on this topic when assessed by ACT. We determined that systems of linear equations have high coherence value, mathematically; that this topic is included by all high performing nations; and that it has moderately high value to college faculty. Result: We included it in our standards.

A different and more complex pattern of evidence appeared with families of functions. Again we found that students performed poorly on problems related to many advanced functions (trigonometric, logarithmic, quadratic, exponential, and so on). Again we found that a number of states cover them, even though college faculty rated them lower in value. High performing countries include this material, but with different degrees of demand. We decided that we had to carve a careful line through these topics so that limited teaching resources could focus where most important. We decided that students should develop deep understanding and mastery of linear and exponential functions. They should also have familiarity with other families of functions, and apply their algebraic, modeling and problem solving skills to them—but not develop in-depth technical mastery and understanding. Thus we defined two distinct levels of attention and identified which families of functions got which level of attention.

Why were exponential functions selected for intensive focus in the Functions standard instead of, say, quadratic functions? What tipped the balance was the high coherence value of exponential functions in supporting modeling and their wide utility in work and life. Quadratic functions were also judged to be well supported by expectations defined under Expressions and Equations.

These examples indicate the kind of reasoning, informed by evidence, that it takes to design standards aligned to the demands of college and career readiness in a global economy. We considered inclusion in international standards, requirements of college and the workplace, surveys of college faculty and the business community, and other sources of evidence. As we navigated these sometimes conflicting signals, we always remained aware of the finiteness of instructional resources and the need for deep mathematical coherence in the standards.

At the end of this document, there is a listing of a number of sources that played a role in the deliberations described above and more generally throughout the process to inform our decisions. A hyperlinked version of the bibliography can be found online at www.corestandards.org.